

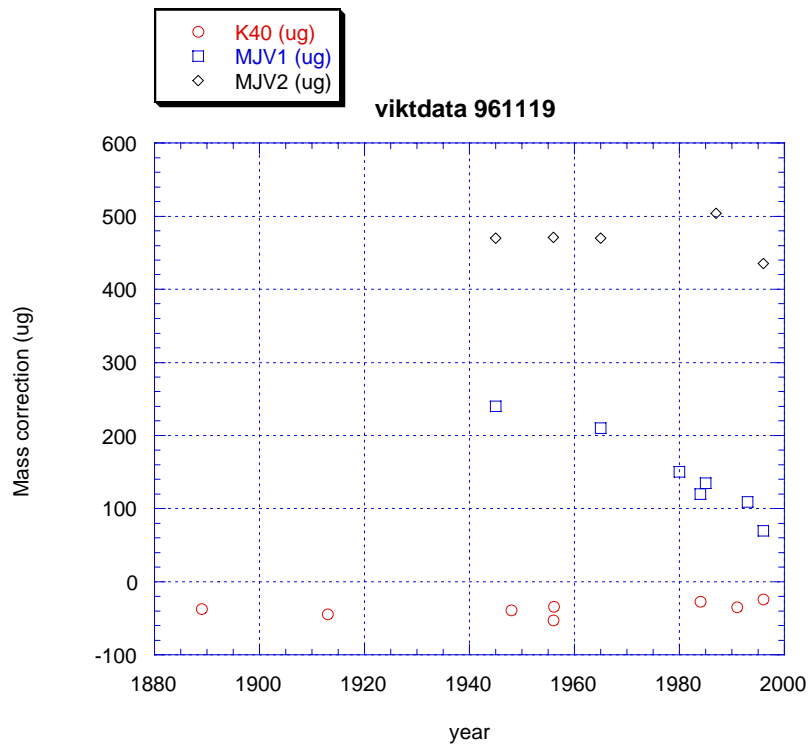
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Bengt Johansson

**11th Comparison  
between the Swedish  
national kilogram and  
SP principal standards  
for one kilogram**

SP Report 1996:50

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## Abstract

11<sup>th</sup> Comparison between the Swedish national kilogram and SP principal standards for one kilogram

Measurements of mass in Sweden are traceable to international standards through the mass of the Swedish platinum-iridium prototype kilogram, **K 40**. This report concerns weighings between the national prototype and (a) the international kilogram at the B.I.P.M in 1991 (3<sup>rd</sup> Periodic Verification of the National Prototypes of the Kilogram (1988 - 1992), and (b) subsequently with various principal standard kilograms, at the Swedish National Testing Institute (SP), Borås.

The weighing comparison at SP, the eleventh of **K 40** with principal standard kilograms since 1894, is the first to be performed on an electromagnetically compensated mass comparator installed in an air tight chamber.

The principal results of the weighings are:

(i) New values for the mass standards **K 40** (Pt-Ir); **F, H** (gilded brass); **MJV1, MJV2, G1, Me** (stainless steel) with respect to the international kilogram;

(ii) Variation of weights with time, in some cases continuing a study of nearly one hundred years (for **K 40, F, H**);

(iii) A detailed study of weighing on an electromagnetically compensated balance in an air tight chamber.

Key words: Kilogram, Sweden, Prototype, Pt-Ir, Mass standard, Comparison, Traceability

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The experienced, practical help of Krister Stolt has been invaluable. Ulf Jacobsson assisted in the evaluation of the results and Lars Börjesson helped with the weighing. Thanks are also due to those who assisted in the calibration of the various instruments employed during the comparisons at SP (see respective appendices).

The National Testing & Research Institute has provided both the laboratory facilities and economic support for the comparisons. Some support of the SWEDAC has also been provided.

## 1. INTRODUCTION

Over the more than one hundred years since Sweden officially adopted the metric system of weights and measures in 1889, traceability to international standards of mass has been maintained through comparison of the prototype kilogram **K 40** with the International Kilogram at the BIPM.

The present work concerns the latest weighing of **K 40**

(a) with the international kilogram at the B.I.P.M, as part of the third periodic verification of the national prototype kilograms, completed in 1992 [Girard 1992, 1994], and

(b) subsequently with various principal standard kilograms at the Swedish National Testing Institute (SP).

The principal standard kilograms of SP involved in the present work are

(i) a gilded brass weight, **H**, which has been weighed against **K 40** at regular intervals over nearly one hundred years;

(ii) two stainless steel weights, **MJV1** and **MJV2**, manufactured in the nineteen forties;

(iii) stainless steel weights, **G1** and **Me**, of relatively recent manufacture and other kilogram standards.

Weighings between **K 40** and the principal standard kilograms have taken place at approximately ten-year intervals over the past hundred years, most recently in 1988 [Pendrell and Källgren 1988]. The present comparison is thus the 11<sup>th</sup> since 1889.

The 11<sup>th</sup> comparison at SP is the first to be performed with an electromagnetically compensated mass comparator (Sartorius C1000S) installed in an air tight chamber at atmospheric pressure. It was necessary to compensate the weighing results for the influences of air buoyancy, particularly since weights of different densities were measured.

The principal results of the weighings are:

(i) New values for the mass standards **K 40** (Pt-Ir); **F**, **H** (gilded brass); **MJV1**, **MJV2**, **G1**, **Me** (stainless steel) with respect to the international kilogram;

(ii) Variation of weights with time, in some cases continuing a study of nearly one hundred years (for **K 40**, **F**, **H**);

(iii) A detailed study of weighing on an electromagnetically compensated balance in an air tight chamber.

This report contains full details of the weighings and results. Comparisons with previously-used routines are given. Somewhat modified weighing procedures are employed, as well as new data acquisition (in *Visual Basic*) and evaluation programs (in *Mathcad*) for a PC computer.

## 2. LABORATORY SET-UP

### 2.1 Comparison laboratory

The primary laboratory for weighings of the highest accuracy is situated in an underground location in a separate building at the perimeter of the Swedish National Testing Institute (SP) in Borås. The laboratory itself is housed in a self-contained construction within the building. All instrument tables in the laboratory are supported, independently of the floor, on a large concrete block. The block rests on the bedrock, cushioned by a rubber mat about 1 cm. thick. In this way, vibrations are minimised. [Pendrill 1993]

### 2.2 Air tight chamber

The air tight chamber housing the mass comparator is shown below. The chamber, made at SP, was placed on an instrument table in the comparison laboratory.

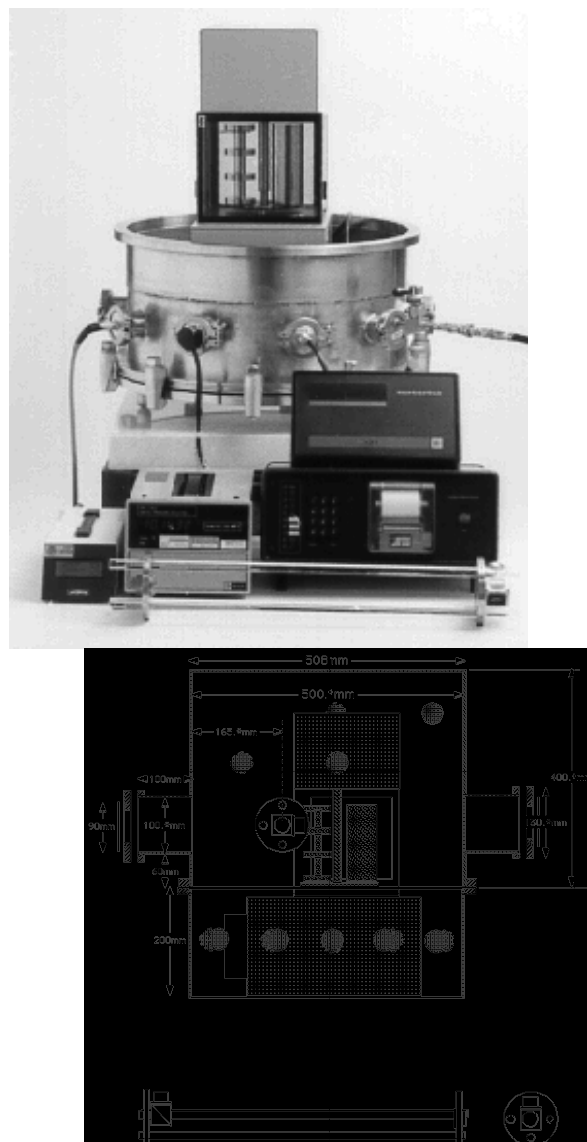


Figure 1 Drawings of the SP air-tight weighing chamber, including laser refractometer and artefacts

The air pressure and temperature in the balance is registered by sensors described in detail in section 4.

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During a typical weighing comparison (of some ten hours' duration) the air pressure drifted by about 10 Pa, mostly due to air temperature changes caused by heat from the mass comparator.

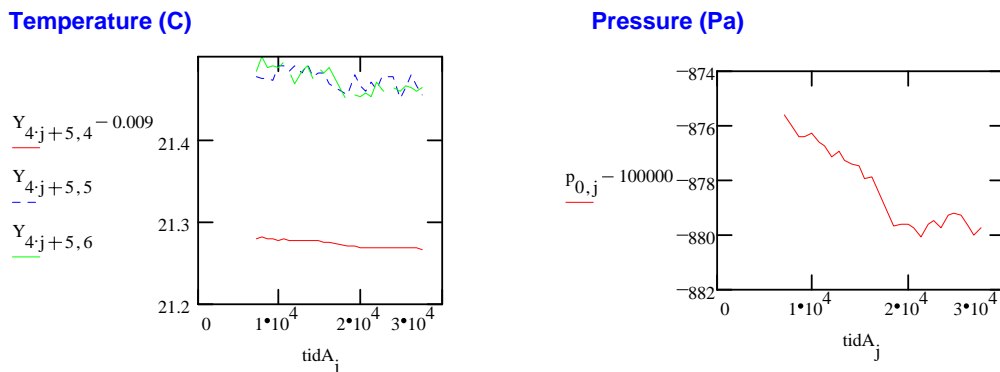


Figure 2 Temperature and pressure in the air-tight chamber during a comparison

### 2.3 Mass comparator

The mass comparator is a commercial machine (Sartorius C1000S) based on the Ångström principle of electromagnetic compensation [Ångström 1895]. The dynamic range of the comparator is between - 100 mg and + 500 mg about 1kg and the resolution 1  $\mu$ g.

The mass comparator generated significant amounts of heat. This necessitated a period of thermal stabilisation of the chamber during which the comparator was operated over a period of about 24 hours prior to weighing. Air temperature in the chamber drifted by about 0,02 °C during a weighing comparison of about 10 hours.

The sensitivity of the comparator could not be determined during automatic operation in the chamber. Instead, the comparator readings at 0, 1 mg, 2 mg and 5 mg were determined before closing and after opening the chamber. The sensitivity was therefore characterised in this work by a relatively large uncertainty ( $\pm 15 \mu$ g).

### 2.4 Mass standards

The present work concerns the latest weighing of **K 40**

(a) with the international kilogram at the B.I.P.M, as part of the third periodic verification of the national prototype kilograms, completed in 1992, and

(b) subsequently with various principal standard kilograms at the Swedish National Testing Institute (SP).

The principal standard kilograms of SP involved are

(i) a gilded brass weight, **H**, which has been weighed against **K 40** at regular intervals over nearly one hundred years;

(ii) two stainless steel weights, **MJV1** and **MJV2**, manufactured in the nineteen forties;

(iii) stainless steel weights, **G1** and **Me**, of relatively recent manufacture and other kilogram standards.

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A number of small additional weights, in the range 1 mg to 50 mg, was employed for approximate equalisation of the nominally kilogram weights; compensation of differences in apparent mass due to air buoyancy (section 2.5); and *in situ* calibration of the sensitivity of the balance.

### 2.4.1 K 40

The prototype kilogram, **K 40**, made of 90% platinum and 10% iridium, is in the form a cylinder of diameter equal to its height, and with a volume, according to the certificate 1993 at 0 °C of 46,411 5 cm<sup>3</sup> [BIPM 1993]. Also mentioned is a coefficient of cubic thermal expansion, used to calculate the volume at a temperature  $t$  (°C), of  $\alpha = (25,863 + 0,005\ 62t)10^{-6}\ \text{K}^{-1}$ .

The prototype kilogram has been weighed at the B.I.P.M. on the occasions of the Periodic Verifications of the National Kilogram Prototypes: in 1889, **K 40** was found to have a weight 1 kg - 0,037 mg; in 1948, 1 kg - 0,039 mg, and most recently, in 1991 (26 February), 1 kg - 0,035(2) mg, relative to the international kilogram.

The calibration certificate issued on this last occasion is reproduced in Appendix A. The certificate is accompanied by a statement of the result of a weighing of **K 40**, before cleaning, 1 kg - 0,016 mg. These results show that, while the mass of **K 40** remains stable within measurement uncertainty limits over more than one hundred years, it is possible for the mass of the kilogram to change by many times the quoted uncertainty limits, owing to some change in the kilogram which may be compensated by the above-described cleaning procedure. As specified in the BIPM calibration certificate, the mass of the prototype is expected to increase in mass by + 0,036 µg per day during the first three months. It is also known that K40, as with the majority of prototype kilograms changes mass after about one year at a rate of + 1 µg/year. Figure 3 shows predicted variation in the mass of K40 based on these observations, and an estimated evolution at an intermediate rate for the intervening period > 3 months and < 1 year after cleaning.

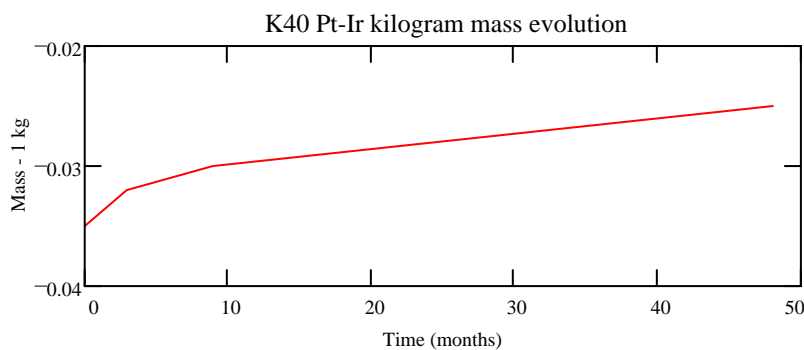


Figure 3 Predicted evolution of mass of kilogram **K40** following calibration at 910226

Further detailed descriptions of the appearance of **K 40**, as well as of the procedures used for storing and for transporting the weight to the balance at various times from directly after returning from the BIPM forward, including the occasions of the three weight comparisons reported in this work, are given in Appendix B.

### 2.4.2 Principal standard kilograms

#### 2.4.2.1 Gilded brass kilogram, **H**.

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The gilded brass kilogram, denoted **H**, is of interest since it has been weighed with **K 40** at all comparisons performed since the first in 1894.

The kilogram **H** has the following volume, determined by hydrostatic weighing by Ekstrand and Ångström [1884]

**H**

volume(12,9°C) = 121,651 ml. volume(0°C) = 121,560 ml = 121,563 cm<sup>3</sup>.

The coefficient of cubic thermal expansion used to calculate the volume of the gilded brass weight at a temperature  $t$  °C is taken to be  $\alpha = 58 \times 10^{-6} \text{ K}^{-1}$ .

Table 2.2 gives the values of the mass of **H** for each comparison.

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**Table 1** Mass of gilded brass kilogram, **H**.

<b>H</b> - 1 kg year	(mg)
1894	+ 6,69(1)
1904	+ 6,67(-)
1914	+ 6,69(3)
1915	+ 6,68(-)
1924	+ 6,75(3)
1935	+ 6,75(3)
1945	+ 6,84(3)
1955	+ 7,00(4)
1965	+ 6,84(3)
1980	+ 6,80(2)
1985	+6,887(16)

Owing to the fact that the density of brass is quite different from that of Pt-Ir, compensation for large air buoyancy effects has been necessary, with associated larger uncertainties in the weighing results, as indicated by the uncertainty limits quoted in parentheses in Table 1. The variation in the mass of **H** over nearly one hundred years, however, is due to other factors as well, such as changes in the surface condition through becoming soiled or through wear during handling.

### 2.4.2.2 Gilded brass kilogram, **F** (Uppsala University)

The brass weight, denoted **F**, belonging to Uppsala University, Institute of Physics, is particularly interesting since it predates all of the other kilograms in this comparison, being acquired by A J Ångström in 1867 from Paris [Ekstrand and Ångström 1884].

The volume of **F** has not been determined with the highest accuracy: Knut Ångström, in connection with the 1<sup>st</sup> comparison in 1894, was not sure whether the knob was screwed on or was an integral part of the kilogram. Hydrostatic weighing was therefore not attempted. Instead, Ångström developed an equipment based on the displacement of air [Ångström 1895], by which he obtained the volume:

[Ekstrand and Ångström 1884]:

$$\frac{\mathbf{F}}{\text{volume}(15^{\circ}\text{C})} = 124,05(5) \text{ ml. } \text{volume}(0^{\circ}\text{C}) = 123,94(5) \text{ ml} = 123,94(5) \text{ cm}^3$$

**F** was weighed at the 1<sup>st</sup> comparison, 1894, and found to weigh 1 kg - 0,00(2) mg.

### 2.4.2.3 Stainless steel kilograms **MJV1** and **MJV2**

The two stainless steel kilograms, denoted **MJV1** and **MJV2**, were manufactured for the 6<sup>th</sup> comparison in 1945 [Grabe *et al.* 1950]. Both weights are in the form of a cylinder of diameter approximately equal to the height, with slightly rounded edges. Each weight is made in one piece, without internal cavities nor pieces added. Each weight has the notation '**MJV1**' or '**MJV2**' inscribed on the top face.

The weights are made of austenitic stainless steel with a composition of 18.2% Cr, 8.5% Ni and 0.08% C, according to an analysis made by Statens provningsanstalt. The coefficient of cubic thermal expansion used to calculate the volume of each stainless steel weight at a temperature  $t$  °C is taken to be  $\alpha = 48 \times 10^{-6} \text{ K}^{-1}$ .

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The kilogram **MJV1** accompanied **K 40** to the BIPM in 1984 /7/ where it was weighed hydrostatically, thereby re-determining the volume to be:

$$\mathbf{MJV1} \quad \text{volume}(0\text{ }^{\circ}\text{C}) = 126.659\ 8(10)\ \text{cm}^3$$

It is assumed that the kilogram **MJV2** also has the same volume.

Magnetic susceptibility measurements made at the PTB reported  $\chi$  values of as high as 0,01 for mass **MJV1** (appendix C). No evidence, however, of magnetic effects on weighings has been found so far.

**Table 2** Masses of stainless steel kilograms **MJV1** and **MJV2**.

Comparison	MJV1 - 1 kg	MJV2 - 1 kg	
year	(mg)	(mg)	
1945	+ 0.24(3)	+ 0.47(3)	
1955	+ 0.31(4)	+ 0.59(4)	
1956 Sept.	–	+ 0.471(-)	BIPM
1956 Nov.	–	+ 0.512(-)	BIPM
1965	+ 0.21(-)	+ 0.47(-)	
1980	+ 0.15(2)	–	
1984	+ 0,120(15)	–	BIPM
1985	+ 0,135(16)	–	
1987	–	+ 0,504(16)	
1993-09-13	+ 0,109(8)	–	PTB

### 2.4.2.4 Stainless steel kilogram **G1**.

The stainless steel kilogram, notated **G1**, was manufactured by the company Gragerts Våg- och viktservice AB in Stockholm in 1974. The weight is in the form of a cylinder of diameter approximately equal to the height, with slightly rounded edges.

The weight is made in one piece, without internal cavities nor pieces added. The weight has the notation '**G1**' inscribed on the top face. The weight is made of acid-proof austenitic stainless steel (Uddeholm) corresponding to DIN 4305 (18% Cr, 10.5% Ni, 2% Mn, 1% Si and 0.15% C).

The kilogram **G1** accompanied **K 40** to the BIPM in 1984, where it was weighed hydrostatically, thereby redetermining the volume to be:

$$\text{volume}(0^{\circ}\text{C}) = 124,578\ 9(10)\ \text{cm}^3$$

The coefficient of cubic thermal expansion used to calculate the volume of the stainless steel weight at a temperature  $t\text{ }^{\circ}\text{C}$  is taken to be  $\alpha = 43,5 \times 10^{-6}\ \text{K}^{-1}$ .

#### Mass of **G1**:

Comparison	G1 - 1 kg	
year	(mg)	
1984	+ 2,350(15)	BIPM
1987	+ 2,399(20)	

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### 2.4.2.5 Stainless steel kilogram Me.

This kilogram was supplied by the Mettler-Toledo Corp as a mass standard intended for OIML accuracy class E<sub>1</sub>. The volume of the kilogram was determined at SP prior to this comparison by hydrostatic weighing and found to be (appendix D):

$$\text{Volume(Me, 0 °C)} = 125,317(6) \text{ cm}^3$$

A calibration performed by Mettler Co. prior to delivery stated a conventional mass of  $1\text{kg} + 0,045(0)\text{mg}$ .

### 2.4.3 Additional weights

#### 2.4.3.1 Additional weights for coarse air buoyancy compensation.

The different volumes of air displaced by the prototype kilogram and the principal standard kilograms described above, cause apparent differences in mass, due to different air buoyancy (see section 4), of about 90 mg between the Pt-Ir kilogram and the other kilograms. Additional leaf weights, in total about 90 mg, were laid on the top face of each kilogram so that the kilogram comparator (section 2.2) displayed similar readings within a few mg for all kilograms. A list of these additional weights together with the results of calibrations, is given in Table 3.

**Table 3 Identification and masses of additional weights for coarse compensation of air buoyancy effects.**

Additional weight	Mass (mg)	Volume (cm <sup>3</sup> )	Calibration
AII10 20mg	19,9966(15)	$1,105 \times 10^{-3}$	SP 941212
AII10 10mg	10,0054(10)	$0,553 \times 10^{-3}$	SP 941212
AII5 20 mg	19,9920(15)	$1,105 \times 10^{-3}$	SP 941212
AII5 10mg	10,0085(15)	$0,553 \times 10^{-3}$	SP 941212
AII5 5mg	5,0138(10)	$1,857 \times 10^{-3}$	SP 941212

#### 2.4.3.2 Sensitivity weight.

The sensitivity of the Sartorius mass comparator is known to be variable, even under relatively stable conditions. It is therefore necessary to determine the sensitivity during each weighing series. The actual procedure adopted in this work is described in section 3. It is sufficient to note here that small sensitivity weights, of nominal mass 1 mg, 2 mg and 5 mg, are in turn alternately loaded or off-loaded from the weighing of each kilogram. The masses of the sensitivity weights have been calibrated recently, with the following results:

**Table 2.5. Mass of sensitivity weight.**

Nominal weight	Mass(mg) calibration 1994-12-12	Mass(mg) calibration (mg) 1995-
1		
2		
5		

## 2.5 Mass standard characterisation

A number of characteristics of mass standards, including density, surface roughness, magnetism etc determine how stable and reliable each standard is.

### 2.5.1 Surface roughness

The total integrated scattering (TIS) of light from the surface of each mass standard over a circular sample area of about 5 mm diameter was determined with an Ulbricht sphere [Pendrill 1995]. The surface roughness parameter  $\sigma$  was related to  $R_z$  ISO (as specified in OIML recommendation R111) by measurements made on a set of surface roughness standards (Microsurf 316) with TIS with previous measurements with a mechanical surface roughness meter (Formtalsurf). The functional relation is shown in figure 4.

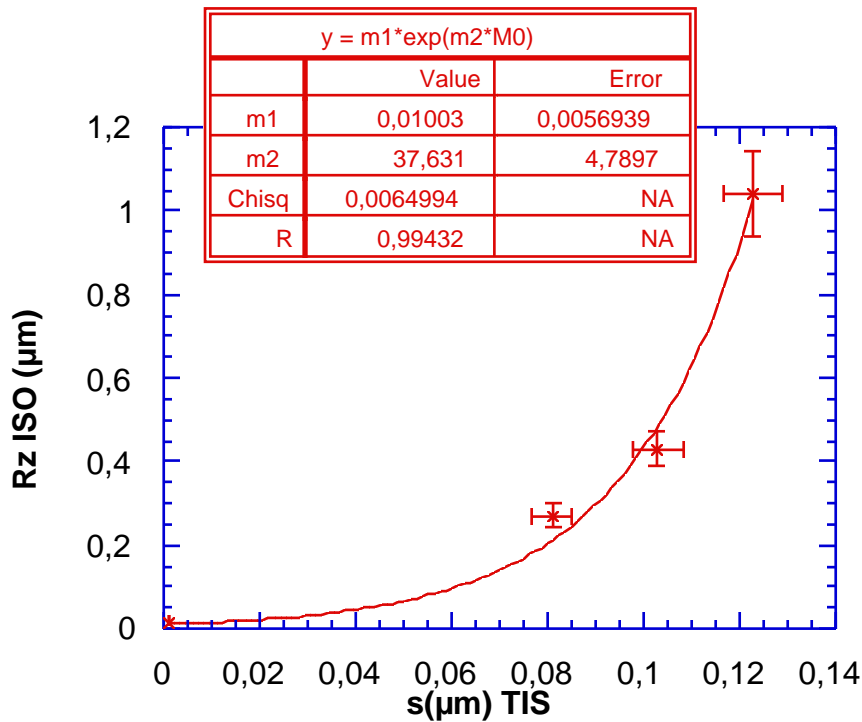


Figure 4 Relation between surface roughness parameters  $R_z$  ISO and  $\sigma$  (TIS)

Table 5 Relation between surface roughness parameters  $R_z$  ISO and  $\sigma$  (TIS)

Object	$\sigma$ TIS (µm)	$R_a$ (µm)	$R_z$ ISO (µm)	$R_z$ ISO calc (µm)
1	0,081(10)	0,030(2)	0,27(3)	0,21(2)
2	0,103(10)	0,047(2)	0,43(4)	0,48(5)
3	0,123(10)	0,114(6)	1,04(10)	1,0(2)
MJV1 u	0,024(5)			0,025(10)
MJV2 u	0,026(5)			0,027(10)
G1 u	0,049(5)			0,063(13)
Me u –	0,046(5)			0,057(13)
Me u //	0,048(5)			0,061(13)
F u	0,123(10)			1,0(2)

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Table 5 gives the measured values of  $\sigma$  and the calculated values of  $R_z$  ISO for several of the mass standards.  $u$  denotes the upper face of each weight. The weight **Me** showed signs of mechanical polishing, resulting in slightly different values of  $\sigma$  TIS parallel (//) and perpendicular (–) to the scratch marks [Pendril 1995]. The OIML R111 value of  $R_z$  ISO is 0,5  $\mu\text{m}$  for class E<sub>1</sub> weights.

### 2.5.2 Magnetism

The magnetic susceptibility,  $\chi$ , of each of SP's principal kilogram mass standards was checked using the internal method MVm 22.1 Using a commercial permeability indicator (Precisa #5917) with accompanying reference material, it was shown that all checked weights satisfied the requirements in OIML Recommendation 111 (Accuracy classification of weights) for standards in class E<sub>1</sub> (the best possible class) with  $\chi < 0,01$ .

Measurements made at the PTB of the magnetic susceptibility,  $\chi$ , of MJV1 (statement dated 93-09-13 in Appendix C) reported locally high values (up to 0,01) at certain parts of the kilogram. There was however no evidence that this affected the weighing result with a series of measurements made on a kilogram comparator of the same kind as used in the present comparison.

## Fel! Okänt växelargument.

### 3. WEIGHING PROCEDURE.

#### 3.1 Preparation for a comparison.

##### 3.1.1 Instrument calibration.

All instruments (section 4) and additional weights (section 2.4.3) were calibrated prior to the comparison. Details of these calibrations are given in the respective sections.

##### 3.1.2 Preparation of weights

The kilograms to be weighed are stored in a vault, under lock and alarm, close to the comparison laboratory (section 2). The prototype normally is placed in a locked cupboard (one of the few items, apart from the prototype, from the nineteenth century) in the vault under 3 glass domes, the weight resting on a fused quartz disc on a piece of chamois leather (appendix B). A hair hygrometer indicates the current humidity in the cupboard, and a thermometer shows both the current temperature in the cupboard, as well as the maximum and minimum temperatures which have occurred since the last time the cupboard was opened. Typical temperature variations are between 17 and 23 °C.

After removal of the three glass domes, the prototype is carefully lifted, with a special pair of tongs (which have always and exclusively accompanied **K 40**, and are stored in the same cupboard) on to the base of a transport holder standing on a table close to the cupboard. Prior to mounting the holder, a careful examination of the prototype is performed, in the presence of a witness, and a protocol is subsequently written (see relevant protocols in appendix B). The transport holder, used whenever the prototype is moved outside of the vault, including shipment to and from the B.I.P.M., has three pads (with chamois leather) which are used to hold the kilogram in place by tightening a screw behind each pad.

The prototype is then carried, in its holder, to a table besides the mass comparator (section 2.2). The holder is removed from the prototype, and the prototype is carefully examined: any changes in the visual appearance of the weight are recorded on the protocol. Any dust or other loose contaminations are gently brushed off, but no extensive cleaning is performed. Using the same tongs, the prototype is lifted, turned horizontal, and brought over to the comparator, where it is returned to upright. The prototype is first lowered onto the weight carousel.

Similar moving procedures are followed when handling the principal standard kilograms, although these weights are usually carried to the balance in the holders in which they are normally stored. The only mass to be cleaned with more than a light brushing was the kilogram **F**, which was carefully wiped clean with an alcohol-soaked chamois leather on the underside.

The balance can accommodate up to 4 kilogram weights.

As described in section 2.4.3.1., additional weights were placed on the top face of the principal kilograms, for coarse compensation of air buoyancy effects relative to the more dense prototype.

Each kilogram to be weighed was then loaded and off-loaded from the scale pan a number of times. In this way, each kilogram became centred on the scale pan.

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The procedure described above was performed several days prior to the planned start of the weighings. Before closing the air tight chamber, the sensitivity of the comparator was determined at three loadings (1 mg, 2 mg and 5 mg) and the actual air density noted. After this, the room containing the balance was not entered until after the weighings were over. Weighings, as performed under the automatic control of a PC computer, were started normally about 24 hours after. In the intervening time, the carousel of the comparator, bearing the 4 kilograms to be weighed, was continuously rotated to achieve thermal equilibrium.

### 3.2. Weighing comparison.

The kilograms were arranged in two groups: those to be weighed directly against the prototype kilogram K40 and those forming a group of secondary standards, as shown in figure 5. The mass standards in the first group were then weighed pair-wise.

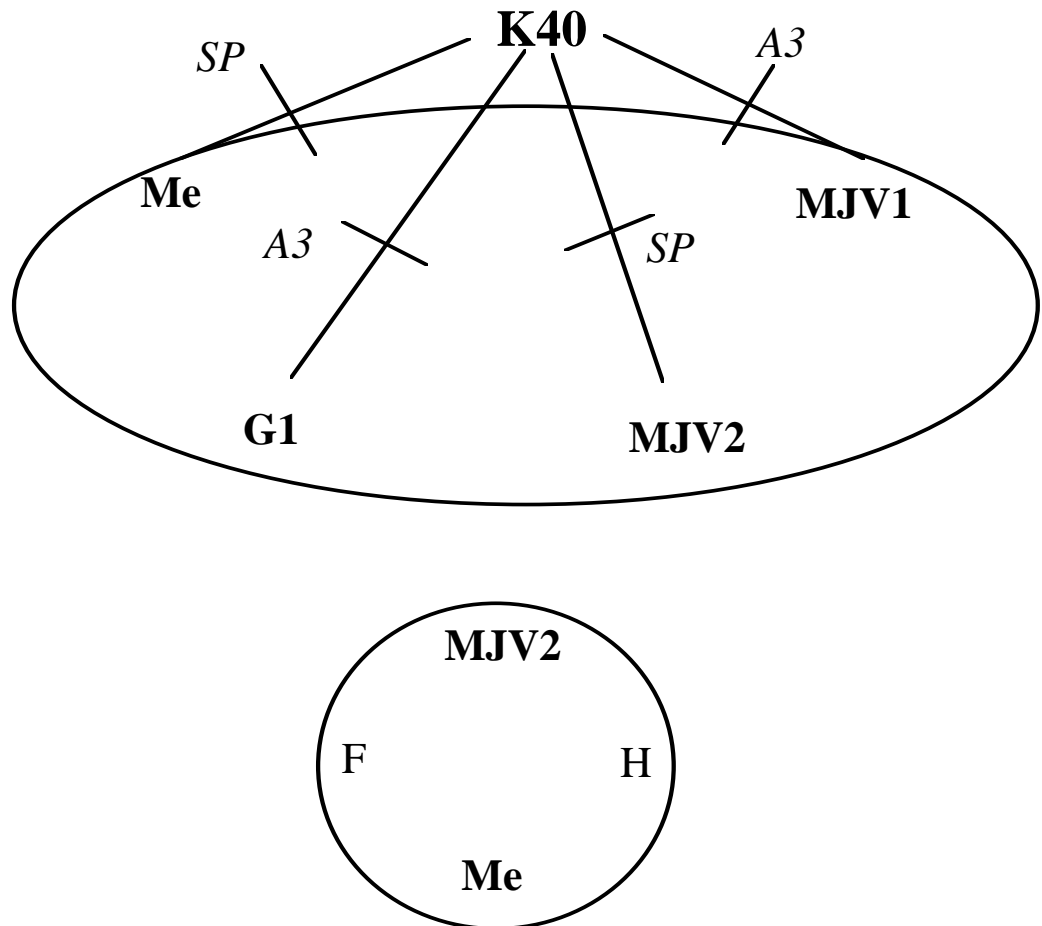


Figure 5 Scheme of the kilogram comparison

#### 3.2.1 Rectangle sequence (i, N; k, N-1)

The weighing comparison consisted of a number of rectangle sequences involving the successive weighing of four kilograms in weighing series described in section 3.2.2. The rectangle sequence was repeated up to about 40 times in all.

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The four weighing differences (or "deflections"), and associated variances, are:

$$\begin{aligned} d_1 &= N_1 - A_1 ; s_{12} \\ d_2 &= A_2 - B_2 ; s_{22} \\ d_3 &= B_3 - C_3 ; s_{32} \\ d_4 &= C_4 - N_4 ; s_{42} \end{aligned} \quad (3.1)$$

where the subscripts indicate that the weight value for any weight may vary from weighing to weighing.

The task of the comparison is to determine the correction,  $c$ , to the mass of each kilogram with respect to the SI kilogram, where 1 kg reference (or "true") value is defined as the mass of the International Kilogram at the BIPM. The weighing equations (3.1) can be expressed in matrix form as

$$\mathbf{d} = \mathbf{A} \cdot \mathbf{c} \quad (3.2)$$

where  $\mathbf{A}$  is the design matrix (see below).

The corrections,  $c_i$ , to the nominal masses of the four kilograms ( $i=1..N-1$ ,  $N=4$ ) are given by the least-squares solution:

$$\mathbf{c} := (\mathbf{A}^T \cdot \mathbf{W} \cdot \mathbf{A})^{-1} \cdot \mathbf{A}^T \cdot \mathbf{W} \cdot \mathbf{d}$$

where the weighting matrix,  $\mathbf{W}$ , is given by the inverse of the observed variances,  $s_i^2$ , in the deflections, normalised by the sum of these variances (excluding that of the restraint, see below):

$$W_{i,i} := \left( \frac{1}{s_i} \right)^2 \cdot \left[ \frac{1}{\sum (s \cdot s)} - (s_0)^2 \right]$$

The single restraint, the known correction,  $c_0$ , to the nominal mass of the standard kilogram, is added as a deflection to the weighing equations (3.2) according to Gauss-Markov procedure, so that the design matrix is

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 1 \end{bmatrix} \quad \begin{array}{l} \text{where, in the weighting matrix, } \mathbf{W}, \text{ the standard is given infinite} \\ \text{(} W_{0,0} := 10000 \text{) weighting.} \end{array}$$

### 3.2.2 A series of weighings (j, M-1)

A series of weighings in the present work consists repeated cycles,  $j := 0 .. (M-1)$ , of weighings (3.2.3) of the four kilograms in order, thus  $(N, A, B, C)_M$ . This circular comparison sequence has one of the better performances for 4 items in compensating for not only linear, but also quadratic and cubic drift [Sutton and Clarkson 1993/94]. The sequence used in earlier comparisons [e.g. Pendrill & Källgren 1988],  $(ABBA)_M$ , is apparently less efficient.

Before starting the weighing, information about the comparison was recorded in a PC datafile (named  $mxx\_xxxx$  with the format  $mdate\_start\ time.$ , for example,  $m06\_2017$  was the datafile of the comparison which started on 950506 at 2017).

The format was as follows:

0	1	2	3	4	5	6	7	8	9 - 12	13
---	---	---	---	---	---	---	---	---	--------	----

## Fel! Okänt växelargument.

1	M	coc	dew	$d_s$	$m_s$	$s(m_s)$	0	start time	$h_k$ (k:0..N-1)	0
2	k	$V0_k$	$\alpha_k$	$\beta_k$	$c_k$	$s(c_k)$	$s(V0_k)$	$Tm_k$	$Vm_k$	0

M = number of cycles

coc = CO<sub>2</sub> concentration

dew = dew-point temp (initial value) ( °C)

$d_s$  = deflection when loaded with sensitivity mass (div)

$m_s$  = mass of sensitivity weight (mg)

$s(m_s)$  = uncertainty of sensitivity mass

$h_k$  = height of centre-of-mass above base of kilogram  $k$  (m)

$V0_k$  = volume of kilogram  $k$  at 0 °C (cm<sup>3</sup>)

$$V_{k,j} := V0_k \cdot \left[ 1 + (\alpha_k + \beta_k \cdot \text{temp}_{k,j}) \cdot \text{temp}_{k,j} \right]$$

$c_k$  = correction to kilogram mass (mg)

$s(c_k)$  = uncertainty in  $c_k$  (mg)

$Tm_k$  = mass of additional mass(es) (mg)

$Vm_k$  = volume of additional mass(es) (cm<sup>3</sup>)

### 3.2.3 A weighing

At each loading of the mass comparator, data was collected by a PC computer in the following order:

0	1	2	3	4	5	6	7	8 - 13
tid <sub>k,j</sub>	visning <sub>k,j</sub>	laser <sub>k,j</sub>	texas <sub>k,j</sub>	temp <sub>k,j</sub>	tempL <sub>k,j</sub>	tempU <sub>k,j</sub>	dew <sub>k,j</sub>	0

tid = time since start (s)

visning = comparator reading (g)

laser = optical path length change registered by laser refractometer ( $\lambda/400$ )

texas<sub>k,j</sub> = reading of barometer (Texas)

temp = temperature 1( °C) -- inside comparator case

tempL = temperature 2( °C) -- at the laser refractometer

tempU = temperature 3( °C) -- at the top of the chamber

dew = dew-point temperature (°C)

and this was stored in a computer file following the header specifying the comparison (3.2.2).

The displayed reading of the comparator was corrected for the effects of air buoyancy (section 4).

## 4. AIR BUOYANCY

In the transfer of the unit of mass through weighing of Pt-Ir prototype kilograms with secondary standards in stainless steel (or other materials), a major source of uncertainty arises in the determination of air buoyancy effects. To compensate for an apparent mass difference,  $\rho_w (V_1 - V_2)$ , in the weighing of two kilograms (of volume  $V_1$  and  $V_2$ ) in air of density  $\rho_w$ , which is typically of the order of 100mg, to an accuracy comparable with present-day balance resolution of several  $\mu\text{g}$ , requires a determination of air density to several parts in  $10^4$  [Giacomo 1982].

Air buoyancy corrections also need to be made in determining the masses of the additional weights laid on the kilograms of stainless steel and brass (section 2.3.3.1) for coarse compensation of air buoyancy.

## Fel! Okänt växelargument.

### 4.1 AIR DENSITY MEASUREMENT AND ESTIMATION

#### 4.1.1 Gas density

The number density  $\rho = V^{-1}$  (per molar volume), of a (non-ideal) gas is a function of pressure,  $p$ , and temperature,  $T$ , according to the equation of state:

$$\frac{p}{RT} = \frac{1}{V} \left[ 1 + \frac{B}{V} + \frac{C}{V^2} \dots \right] \quad (1)$$

where the "compressibility"  $Z$  of the gas is expressed as a series of virial coefficients,  $B$ ,  $C$ , ...; and where  $R$ ,  $= 8,314510(70) \text{ J K}^{-1} \text{ mol}^{-1}$ , is the molar gas constant [Giacomo 1982, Davis 1992]. For a gas consisting of several components of relative concentration  $x_i$  of gas with molecular mass  $M_i$ , the mass density,  $\rho_w$ , of the gas is:

$$\rho_w = \frac{p}{ZRT} \cdot \sum_i x_i M_i \quad (2)$$

A weight of mass  $m_0$  (in vacuum) and volume  $V$  displaces an amount of air of density  $\rho_w$  which exerts a buoyancy force equivalent to an apparent loss in mass of  $\rho_w \cdot V$ . The volume  $V$  of the weight is normally determined either hydrostatically (weighing in water or other liquid of known density) or geometrically.

#### 4.1.2 CIPM Recommendations and the Gas Equation of State

The International Committee of Weights and Measures (CIPM) recommends that air density be calculated with an empirical formula [Giacomo 1982, Davis 1992]. The recommendation is based on the gas equation of state (equation (2)) and on a model of standard air.

The model consists of assuming certain relative concentrations,  $x_i$ , of the different constituent gases, including for example that a change in  $O_2$  concentration is always balanced (for example, by combustion) by an equal and opposite change in  $CO_2$  concentration. Measurements are made of air pressure, air temperature and where practical, of constituent gas composition (most often constituent water vapour and carbon dioxide). The presence of, for example, foreign gases would lead to incorrect air density values. The formula has recently been updated [Davis 1992] to include recent changes in some parameters as well as to bring the formulation in line with the new international temperature scale ITS-90.

#### 4.1.3 Measurement of air parameters

Measurement of the air pressure,  $p_{k,j}$ , in the balance was performed by connecting a cm. diameter tube into the lower part of the chamber. The tube, about 3 metres long, was connected at its other end to a precision pressure gauge, of Bourdon coil type (Texas Inst. model 2908). The pressure gauge has been calibrated at a number of occasions during the period of the comparison, the latest calibration certificates (from FFA Swedish National Measurement Laboratory for Pressure, Bromma, Sweden) are reproduced in Appendix E.

The air temperature,  $temp_{k,j}$ , was measured at three locations in the balance -- close to the weight being weighed, and in the vicinity of the laser refractometer and at the top of the chamber -- with three Pt 100 resistance thermometers (Systemteknik AB), calibrated at the Swedish national measurement laboratory for temperature at SP: the calibration certificates are reproduced in appendix F.

## Fel! Okänt växelargument.

The only components of the air which were individually monitored were the air humidity and the concentration of carbon dioxide:

The air humidity, characterised by the dew-point temperature,  $dew_{k,j}$ , was measured by a Protimeter hygrometer installed in the balance chamber. The hygrometer was most recently calibrated at the PTB (the calibration certificate is reproduced in Appendix G).

The carbon dioxide concentration,  $coc_{k,j}$ , in the laboratory air was measured by monitoring the absorption of light at an infra-red wavelength in resonance with a particular transition line in the  $CO_2$  molecule with a spectroscope. The amount of absorption for a known quantity of  $CO_2$  gas was determined prior to each series of weighing. A specification of the instrument is reproduced in Appendix H.

The above measurements of air pressure, temperature and humidity were transferred to the PC computer via interfaces with the respective instruments when the computer was triggered to do so at appropriate times during the comparison (section 3.2.3). The measurement of carbon dioxide was made visually at the beginning and end of each weighing series (section 3.2.2).

## Fel! Okänt växelargument.

### 4.1.4 Calculation of air density

The following is an extract from the Mathcad program calculating the air density from equations based on the equation-of-state, (equation (2)). Absolute temperatures are denoted by the trailer  $T$ , and  $Ma$  is the mass of air.

#### Air density

#### Water vapour

$$A := 1.237884710^{-5} \quad B := -1.912131610^{-2} \quad C := 33.9371104710^0$$

#### Enhancement factors

$$f_{k,j} := 1.00062 + p_{k,j} \cdot 3.14 \cdot 10^{-8} + (\text{dew}_{k,j})^2 \cdot 5.6 \cdot 10^{-7} \quad f1_{k,j} := 1.00062 + p_{k,j} \cdot 3.14 \cdot 10^{-8} + (\text{temp}_{k,j})^2 \cdot 5.6 \cdot 10^{-7}$$

#### Saturated vapour pressures in Pa

$$\text{psv}_{k,j} := e^{\left[ A \cdot (\text{dew}_{k,j})^2 + B \cdot \text{dew}_{k,j} + C + \frac{D}{\text{dew}_{k,j}} \right]}$$

$$\text{psv1}_{k,j} := e^{\left[ A \cdot (T_{k,j})^2 + B \cdot T_{k,j} + C + \frac{D}{T_{k,j}} \right]}$$

$$Xv_{k,j} := f_{k,j} \cdot \frac{\text{psv}_{k,j}}{p_{k,j}}$$

#### Compressibility of air

$$y1_{k,j} := 1.58123 \cdot 10^{-6} - 2.9331 \cdot 10^{-8} \cdot \text{temp}_{k,j} + 1.1043 \cdot 10^{-10} \cdot (\text{temp}_{k,j})^2$$

$$y2_{k,j} := Xv_{k,j} \cdot (5.707 \cdot 10^{-6} - 2.051 \cdot 10^{-8} \cdot \text{temp}_{k,j})$$

$$y3_{k,j} := (Xv_{k,j})^2 \cdot (1.9898 \cdot 10^{-4} - 2.376 \cdot 10^{-6} \cdot \text{temp}_{k,j})$$

$$Z_{k,j} := 1 - \frac{p_{k,j}}{T_{k,j}} \cdot (y1_{k,j} + y2_{k,j} + y3_{k,j}) + \left( \frac{p_{k,j}}{T_{k,j}} \right)^2 \cdot [1.83 \cdot 10^{-11} - (Xv_{k,j})^2 \cdot 0.765 \cdot 10^{-8}]$$

$$Ma_{k,j} := [28.9635 + 12.011 \cdot (\text{coc}_{k,j} - 0.0004)]$$

#### Air density in kg/m3

$$\text{dens}_{k,j} := \frac{p_{k,j}}{Z_{k,j} \cdot R \cdot (T_{k,j})} \cdot \left[ 1 - Xv_{k,j} \cdot \left( 1 - \frac{18.015}{Ma_{k,j}} \right) \right] \cdot Ma_{k,j} \cdot 10^{-3}$$

### 4.2 Air buoyancy corrections to weighing

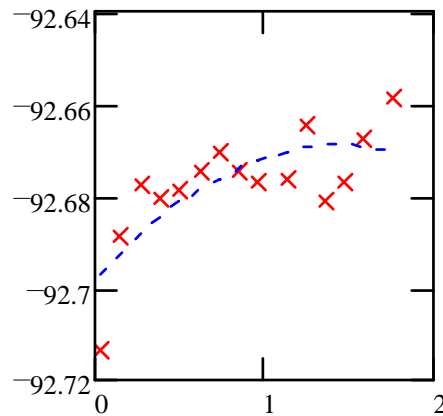
Each comparator reading ( $Y_{4,j+5+k,1}$ ) was corrected for air buoyancy with reference to a "counterweight" of volume  $125 \text{ cm}^3$ .

$$\text{vikt}_{k,j} := \left[ Y_{4,j+5+k,1} \cdot 10^3 - \text{diff}_{k,j} + (V_{k,j} - 125.0) \cdot \text{dens}_{k,j} \right] - (Tm_k - Tv_k \cdot \text{dens}_{k,j})$$

## 5 COMPARISON CALCULATION PROCEDURE.

In order to calculate the difference (or deflection),  $d$ , in mass between pairs of kilograms, the comparator display,  $\text{vikt}$ , (section 4.4) was plotted as a function of time for each kilogram, as shown for example in figure 5.1:

**Fel! Okänt växelargument.**



**Figure 5.1** Comparator reading *vikt* (corrected) (mg) versus time (hours) over a kilogram comparison (m17\_1922 K40 - MJV1) when weighing the Pt-Ir kilogram K40

A least-squares fit was then made to the variation for each kilogram with time to a second-order polynomial:

$$F(t, u) \equiv \begin{bmatrix} u_0 + u_1 \cdot t + u_2 \cdot t^2 \\ 1 \\ t \\ t^2 \end{bmatrix} \quad PA := \text{genfit}(\text{tidA}, \text{viktA}, \text{vg}, F)$$

yielding, for kilogram A (K40) in the comparison m17\_1922 K40-MJV1 the following function F parameters (in units of mg, mg·s<sup>-1</sup> and mg·s<sup>-2</sup>, respectively):

$$PA = \begin{bmatrix} -92.69819 \\ 0.000011 \\ -1.067829 \cdot 10^{-9} \end{bmatrix}$$

The average mass difference between two kilograms, A and B, over the M weighings is then evaluated by taking the difference between fitted functions for the kilograms evaluated at the average time between each weighing pair:

$$d_1 := \frac{1}{M} \cdot \left[ \sum_{j=0}^{M-1} \left( F(\text{tidAB}_j, PA)_0 - F(\text{tidAB}_j, PB)_0 \right) \right] + \delta g \cdot (h_0 - h_1)$$

where

**Fel! Okänt växelargument.**

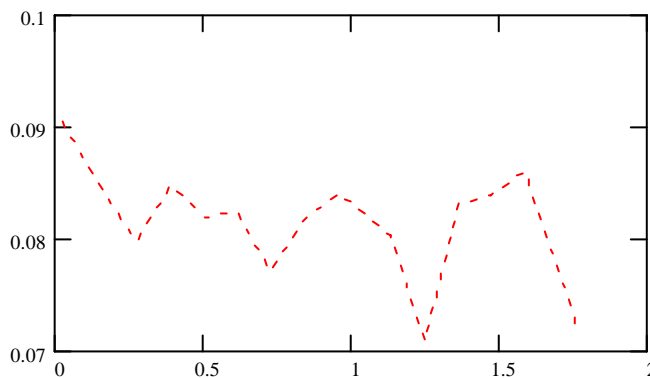
$$\text{tidAB}_j := \frac{\text{tidA}_j + \text{tidB}_j}{2}$$

Note that the mass difference is corrected for the difference in gravitational acceleration:

$$\delta g := -3 \cdot 10^{-7} \cdot 10^6$$

at the different heights,  $h$ , of the centres of mass of the two kilograms.

As an additional check, plots were made of the differences between individual pairs of weighings:



**Figure 5.2** Differences in mass (mg) versus time (hours) between kilograms K40 and MJV1 over a kilogram comparison (m17\_1922 K40 - MJV1)

## 6 DISCUSSION OF ERRORS, UNCERTAINTIES AND CORRECTIONS

### 6.1 'Type A' evaluation of uncertainties

#### 6.1.1 Weighing standard deviation

The observed standard deviations,  $s$ , in the means of the weight differences, were evaluated from the residuals of the least-squares fit for each kilogram of second-degree polynomial  $F(t,u)$  to the corrected comparator readings  $vikt$  versus time, with  $M.(M-3)$  degrees of freedom:

#### m06\_2017 MJV2-MJV1-Me-G1

$$s_1 := \sqrt{(sA^2 + sB^2)}$$

$$s = \begin{bmatrix} 0.023 \\ 0.000948 \\ 0.001229 \\ 0.001001 \\ 0.000625 \end{bmatrix}$$

## Fel! Okänt växelargument.

where

$$s_A := \sqrt{\left| \frac{1}{(M-3)} \right| \cdot \frac{1}{M} \cdot \left[ \sum_j \left( F(\text{tid}A_j, PA)_0 - \text{vikt}A_j \right)^2 \right]}$$

These were mostly of the order of 0,3 - 0,5 mg.

### 6.1.2 Least-squares solution of rectangle of mass differences

Associated with the regression to the rectangle comparison of the four kilograms (section 3.2.1) is the variance-co-variance matrix:

$$V_c := (A^T \cdot W \cdot A)^{-1} \cdot \left[ \sum (s \cdot s) - (s_0)^2 \right] \qquad sc_k := \sqrt{V_{C_{k,k}}}$$

For the comparison

**m06\_2017 MJV2-MJV1-Me-G1**

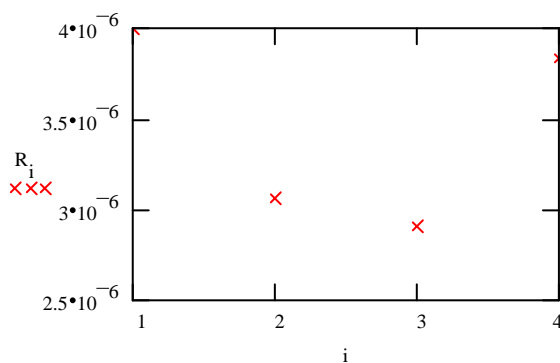
**BIPM**

$$c = \begin{bmatrix} 0.4758 \\ 0.111 \\ 0.6259 \\ 2.3947 \end{bmatrix} \qquad sc = \begin{bmatrix} 0.00083 \\ 0.00002 \\ 0.00095 \\ 0.00092 \end{bmatrix}$$

A third measure of type A uncertainties is got from the observed residuals of the least-squares fit to the rectangle of kilogram comparisons:

### Residuals (mg)

$$R := A \cdot c - d$$



$$gv := \sum (R \cdot R) \cdot \frac{1}{M - N}$$

$$\sqrt{gv} = 0.000001$$

$$M = 28$$

$$N = 4$$

## Fel! Okänt växelargument.

where the group variance,  $g_v$ , may be calculated with a corresponding number of degrees of freedom equal to  $M-N$ , for the  $M$  weighing pairs amongst  $N$  weights.

### 6.2 Type B evaluation of uncertainties

#### 6.2.1 Comparator resolution

The finite resolution of the mass comparator,  $del = 1$  mg, leads to an uncertainty component which here is modelled by a rectangular *a priori* distribution of width  $del$ . As a result, the total variance-covariance associated with the comparator reading was taken as:

$$\Psi_d := V_c + \frac{del^2}{12} \cdot I + g_v$$

where  $I$  is the identity matrix.

$$\Psi_d = \begin{bmatrix} 8.338254 \cdot 10^{-8} & 4.920228 \cdot 10^{-11} & 4.920228 \cdot 10^{-11} & 4.920228 \cdot 10^{-11} \\ 4.920228 \cdot 10^{-11} & 1.710504 \cdot 10^{-7} & 8.295196 \cdot 10^{-8} & 5.235987 \cdot 10^{-8} \\ 4.920228 \cdot 10^{-11} & 8.295196 \cdot 10^{-8} & 1.812016 \cdot 10^{-7} & 6.177191 \cdot 10^{-8} \\ 4.920228 \cdot 10^{-11} & 5.235987 \cdot 10^{-8} & 6.177191 \cdot 10^{-8} & 2.055309 \cdot 10^{-7} \end{bmatrix}$$

#### 6.2.2 Comparator sensitivity

The variance-covariance of the sensitivity was calculated from:

$$\Psi_s := \Psi_d + I \cdot 0.015^2$$

including the uncertainties in the sensitivity determinations, and in the sensitivity mass,  $m_s$ . Even the uncertainty in the comparator reading enters here once again.

$$\Psi_s = \begin{bmatrix} 0.00025 & 4.920228 \cdot 10^{-11} & 4.920228 \cdot 10^{-11} & 4.920228 \cdot 10^{-11} \\ 4.920228 \cdot 10^{-11} & 0.000282 & 0 & 0 \\ 4.920228 \cdot 10^{-11} & 0 & 0.000338 & 0 \\ 4.920228 \cdot 10^{-11} & 0 & 0 & 0.000916 \end{bmatrix}$$

#### 6.2.3 Gravity correction

Uncertainties in the gravity gradient,  $\delta g$ , and in the centre-of-mass heights,  $h$ , are included:

$$\Psi_{g_{k,k}} := 2 \cdot \delta g^2 \cdot u_h^2 + 2 \cdot (h_k)^2 \cdot u_{\delta g}^2$$

#### 6.2.4 Standard kilogram

From the calibration certificate of the kilogram chosen as a standard

## Fel! Okänt växelargument.

$$\Psi_n := (s_0)^2 \cdot I$$

### 6.2.5 Air buoyancy

#### 6.2.5.1 Air density - CIPM method

In discussing the uncertainties associated with the determination of air density with the CIPM formula, a distinction is made between uncertainties due to the formula itself and uncertainties due to measurement of the different quantities, such as pressure, temperature when the formula is applied [Giacomo 1982].

The formula itself is associated with random uncertainties ( $2\sigma$ ) which amount to an average of  $2,5 \times 10^{-5}$  while systematic uncertainties average  $10 \times 10^{-5}$ . Taking typical values for the uncertainties of the various measuring instruments (pressure  $\pm 7$  Pa; temperature  $\pm 10$  mK dew-point temperature  $\pm 0,2$  K), the air density under usual conditions has an associated uncertainty ( $k=2$ ) of  $13 \times 10^{-5}$ .

One should bear in mind that an uncertainty of  $10^{-4}$  in air density would lead to an uncertainty of  $10 \mu\text{g}$  in the mass of a stainless steel kilogram weighed in air against a Pt-Ir prototype.

Calculation of air density with the CIPM method (section 4.1) is associated with uncertainties arising from the formula itself [Giacomo 1982]:

$$\text{uncertainty in BIPM formula } u_{\text{form}} := \frac{10^{-4}}{2} \quad \text{Procès-Verbaux du CIPM, 49, 1981, C1 -15}$$

as well as from uncertainties in the readings of each of the air parameter instruments:

pressure (Pa)	temperature (K)	dew point (K)	CO <sub>2</sub>
$u_p := 7$	$u_t := 0.005$	$u_{\text{dew}} := 0.2$	$u_{\text{co2}} := 0.00005$
$\delta p_p := 1.2 \cdot 10^{-5}$	$\delta p_t := -4.8 \cdot 10^{-3}$	$\delta p_{\text{dew}} := -3.6 \cdot 10^{-4}$	$\delta p_{\text{co2}} := 0.5$

which are combined as:

$$\Psi_{\rho_{k,k}} := u_p^2 \cdot \delta p_p^2 + u_t^2 \cdot \delta p_t^2 + u_{\text{dew}}^2 \cdot \delta p_{\text{dew}}^2 + u_{\text{co2}}^2 \cdot \delta p_{\text{co2}}^2 + u_{\text{form}}^2 \cdot \rho_{k,k} \cdot \rho_{k,k}$$

$$\text{diag}(XV) \cdot \Psi_{\rho} \cdot \text{diag}(XV) = \begin{bmatrix} 0.000118 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0.00012 & 0 \\ 0 & 0 & 0 & 0.000492 \end{bmatrix}$$

where  $\text{diag}(XV)$  is a matrix with the diagonal elements of  $XV$ :

$$XV := (B \cdot \text{Vol})$$

and  $B$  is the design matrix (excluding the restraint).

#### 6.2.5.4 Kilogram volumes

## Fel! Okänt växelargument.

In addition to uncertainties in the hydrostatic density determinations, one must also include uncertainties associated with the thermal expansion coefficients,  $a$  and  $b$ , as well as the temperature measurement:

$$\Psi_{V_{k,k}} := (uVQ_k)^2 \cdot \left[ 1 + \left( \alpha_k \cdot \text{temp}_{k, \frac{M}{2}} \right)^2 + \left( \beta_k \right)^2 \cdot \left( \text{temp}_{k, \frac{M}{2}} \right)^4 \right] + (u\alpha_k)^2 \cdot \left[ (Vol_k)^2 \cdot \left( \text{temp}_{k, \frac{M}{2}} \right)^2 \right]$$

$$\Psi_{V_{k,k}} := \Psi_{V_{k,k}} + (ut)^2 \cdot \left[ \left( \alpha_k \right)^2 + \left( \beta_k \right)^2 \cdot 4 \cdot \left( \text{temp}_{k, \frac{M}{2}} \right)^2 \right] \cdot (Vol_k)^2 + (u\beta_k)^2 \cdot \left[ Vol_k \cdot \left( \text{temp}_{k, \frac{M}{2}} \right)^2 \right]^2$$

$$\rho \cdot B \cdot \Psi \cdot V \cdot B^T \cdot \rho = \begin{bmatrix} 0.000002 & -0.000002 & 0 & -1.309483 \cdot 10^{-12} \\ -0.000002 & 0.000003 & -0.000001 & 0 \\ 0 & -0.000001 & 0.000004 & -0.000003 \\ -1.309483 \cdot 10^{-12} & 0 & -0.000003 & 0.000003 \end{bmatrix}$$

### 6.3 Combined uncertainties

Combining all uncertainty components as a sum of (effective) variances:

$$\Psi_c := \Psi_d + \Psi_s + \text{diag}(XV) \cdot \Psi_\rho \cdot \text{diag}(XV) + \rho \cdot B \cdot \Psi \cdot V \cdot B^T \cdot \rho + \Psi_n + uTm \cdot uTm + \Psi_g$$

$$\Psi_c = \begin{bmatrix} 0.000667 & 0 & 0.000004 & 0.000004 \\ 0 & 0.001041 & 0.000002 & 0.000004 \\ 0.000004 & 0.000002 & 0.001358 & -0.000007 \\ 0.000004 & 0.000004 & -0.000007 & 0.001244 \end{bmatrix} \quad u_{c_k} := \sqrt{\Psi_{c_{k,k}}}$$

## 7 Results and conclusion

The comparison may be summarised with a table of values of the principal kilogram standards determined with reference to the newly-calibrated national prototype kilogram **K40**.

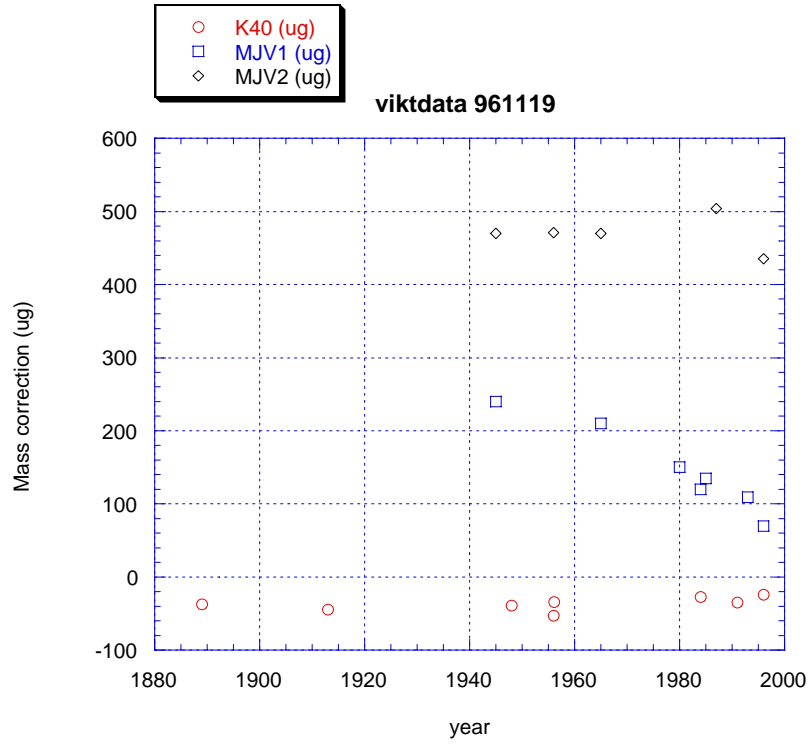
Mass standard	Mass - 1kg (mg)	Date
K40	- 0,035 (3)	9102 BIPM
K40	- 0,028 (8)	9604 BIPM corrected value
MJV1	+ 0,078(20)	9604 SP
MJV2	+ 0,442(20)	9604 SP
G1	+ 2,380()	9506 SP
Me	+ 0,585(26)	9604 SP
H	+ 6,844()	9506 SP
F	- 0,153()	9506 SP

The evolution of some of SPs principal kilogram standards is presented graphically (figure XX).

## Fel! Okänt växelargument.

One result is the clear indication that the kilogram MJV1 is unstable, changing by about  $-5 \mu\text{g}/\text{year}$ .

The determination of the mass of kilogram **F** indicates a change in mass of  $-1,5 \mu\text{g}/\text{year}$ , which is not an unusually large rate of change.



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