Implementation of an Earthquake Test, specified by a Given Response Spectrum

Abstract

A description is given of one way to implement an earthquake test where the test severities are given by response spectra. The test is done by using a biaxial computer aided servohydraulic test rig. The test rig, excitation signal generation, measurement, analyses and presentation of results are described.


1. Introduction

In power plants and other processing industries ensuring safe shutdowns is necessary in case of a serious disturbance, as an earthquake. It is also important that vital parts of the community are intact after such an event. This implies that equipment such as control consoles, battery racks, high voltage equipment and telecommunication equipment must have a granted function for ground vibrations corresponding to the "worst possible" earthquake. The ground motion of an earthquake can be amplified or attenuated in foundation mounted equipment. For a given ground motion, the alteration depends on the system’s natural frequencies and the damping mechanism. For equipment mounted on structures the ground motion is filtered by the building and the secondary structures. The dynamic response of equipment mounted on structures may be amplified or attenuated to an acceleration level many times more or less than the maximum ground acceleration. It is well known that earthquakes are random events and cannot be predicted in detail. Simulating seismic loads by using random waveforms is common. These wave forms can be described by one of the following functions:

(i) Response spectrum
(ii) Time history
(iii) Power Spectral Density, PSD

From now only the response spectrum will be considered. It is a well-established method in earthquake engineering. The response spectrum is a plot of the maximum response, as a function of oscillator frequency of an array of Single Degree of Freedom, SDOF, damped oscillators subjected to the same base excitation. In earthquake engineering the resonance frequencies are in the range 1-40 Hz. The damping can be different in different tests, but a typical value is 5%. If the damage during an earthquake only depends on the maximum response of the test object, the response spectrum describes the damage for test objects modeled as SDOF systems. The response spectrum contains information of the frequency content and the peak acceleration but does not supply information of the actual wave form and its duration.

During the testing the test object is subjected to a multifrequency excitation signal covering the frequency range which can be obtained in a real application. If the test
object is to be mounted in a building the amplification of the building and possible floor mounted structures are taken into account. The response spectrum of the time history measured at the vibrator table is called the Test Response Spectrum, TRS. There is one TRS for each test direction. The obtained TRS:a should exceed the Required Response Spectra, RRS:a, given in the test specification. It is important that the same damping is used in the TRS:a and the RRS:a. The RRS:a can be obtained from standards or by detailed calculations of the responses of buildings and secondary structures. It is then essential to note whether the RRS:a give a general seismic qualification or a if they are valid only for a particular application. Figure 1 shows an RRS with a narrow band peak slightly less than 10 Hz. This peak can be due to a resonance frequency of the building where the equipment is to be mounted.

![Example of a Required Response Spectrum, RRS](image)

**Figure 1:** A narrowband Required Response Spectrum, RRS.

A general seismic qualification is commonly wanted. In such cases broad band RRS:a are demanded. Such spectra have typical strong parts in the frequency range between 2 and 16 Hz. That frequency width is much greater than a typical narrowband region for each specific application. It can be mathematically shown that time histories fulfilling artificial broadened spectra have higher ZPA-values than real narrowband spectra. Using broad band spectra also implies risks for damage or malfunctions due to interactions between different critical frequencies of the tested unit. Such interactions do not occur during a real narrowband earthquake. Using broadened RRS:a sometimes implies overtesting. To avoid overtesting some standards suggest that series of tests with different narrowband spectra covering the whole broad band frequency region can be done.

At sites where earthquakes occur frequently, a lot of smaller earthquakes causing low cycle fatigue damage can precede the "worst possible earthquake." In such case seismic ageing must be done before the test run at the qualification level. This is done by running a number of tests simulating Operating Basis Earthquakes, OBE, before running the test simulating the Safe Shutdown Earthquake, SSE. Typically five OBE tests at 60% of the SSE level are required.
So, the problem is an implicit one: To give the test items a motion that produces a prescribed TRS, i.e., one that conforms as closely as possible with the RRS. The test rig is described in section 2, excitation signal generation in section 3, measurements in section 4, analysis in section 5 and result presentation in section 6.

Besides the qualification test with multiple frequency motion a seismic test often contains a Vibration Response Investigation, VRI. In some standards this test is called a resonance search or exploratory test. The aim of the test is to determine if the test object has any resonance frequencies in the earthquake frequency range. The test should be run at such low level that the test object suffers no mechanical damage. As excitation either noise or sine sweep signals can be used. Often the VRI is repeated after the qualification test. If the test object has suffered global mechanical damage, its resonance frequencies will be lower.

2. Test rig

The principle of the two-axis vibration table at the Swedish National Testing and Research Institute is illustrated in Figure 2. The table is supported on three vertical actuators and the horizontal thrust is provided by a single horizontal actuator arranged as shown in the figure. The dimension of the table is 1.2 · 1.2 m. Due to the three vertical actuators the table can react large bending moments and extending it with beams is possible. The table can be used for tests with simultaneous vertical, horizontal and rotational motion. The dynamic capacity of the table is shown in Figures 3 and 4.

![Figure 2: Schematic sketch of the biaxial test rig.](image-url)
Each actuator is servo controlled with acceleration and displacement feedback by a digital control system, INSTRON 8580. Transfer functions of servohydraulic equipment are always non flat, i.e., high frequencies are damped more than low. Before using a wave form as a drive signal it must therefore be adjusted. This is done by a special software package, called PROFILE CORRECTION, supplied by INSTRON. Before the testing the adjustment is done. The transfer function of the rig is determined when the table is run without any test object mounted on it. If a heavy object is to be tested, a dead weight can be placed on the table. This software can also compensate for unwanted geometric displacement caused by angular movement of the actuators.
3. Excitation signals

There are various ways to achieve an excitation fulfilling the requirements of a broadband frequency signal.

The signals used at the Swedish National Testing and Research Institute are generated as sums of harmonic functions with different frequencies in the range 1 - 40 Hz according to Equation (1).

\[ a(t) = \sum_i A_i \sin(\omega t + \varphi_i) \cdot \Psi(t) \]  

The angular frequencies, \( \omega_i \), are spaced at 1/6 octaves. This ensures that there exist excitation within the 3-dB points of any possible resonant frequency with a damping higher than 2%. The amplitudes, \( A_i \), of the different frequency components are chosen according to the RRS. By an iterative procedure achieving a time-history with a TRS just enveloping the RRS is possible. The phase angles, \( \varphi_i \), are chosen randomly. To get a signal with finite duration the signal is multiplied with a time window function, \( \psi(t) \). This function also gives the time history a smooth start up and decay. The time histories are stored in the computer. Amplitude scaling is possible between different test run in order cover the whole possible range of amplitudes.

4. Measurements

Servo accelerometers are used for measuring the acceleration of the vibrator table. The measurement chain, for one accelerometer, is shown in Figure 5.

![Measurement chain diagram](attachment:measurement_chain_diagram.png)

*Figure 5: The measurement chain for acceleration measurement. Item no: 1 is an amplifier, item no: 2 is an analogue anti alias filter, item no: 3 is a sample and hold unit and item no: 4 is an A/D converter. The sampled signals can by software programs be resampled at other sampling rates and filtered digitally, showed by item no: 5 in the sketch.*
During an earthquake test of a structure it is often required to monitor the dynamic behavior of the test object. For this purpose accelerometers, strain gauges and displacement transducers are mounted on the test object. The control console of the vibrator table contains a data acquisition system for sampling of up to eight external channels. Figure 6 shows a schematic sketch of the data acquisition system.

![Schematic sketch of the data acquisition system.](image)

**Figure 6:** The measurement chain for acceleration measurement. Item no: 1 is the eight input channels, item no: 2 is an amplifier with adjustable sensitivity. By the analogue anti alias filter, item no: 3, the sampled signals are low pass filtered before they are applied to a sample and hold unit, item no: 4. Item no: 5 is a multiplexer and item no:6 is the A/D converter. The sampled signals can by software programs be resampled at other sampling rates and filtered digitally, showed by item no: 7 in the sketch.

By the data acquisition system the analogue signals are low pass filtered for frequencies below 1 kHz and sampled at 5 kHz. As the frequency contents of the signals is less than 50 Hz, the data are by software programs resampled at 200 Hz before long time storage on the disk. The number of data is then reduced without losing any information.

5. Analysis

5.1 Response spectrum analysis

The method used can be summarized as follows: First find the acceleration response of the SDOF systems of interest to the acceleration signal used. The SDOF systems may be described by an impulse response according to Equation (2).

$$a(t) = \omega e^{-\zeta t} \left[ 2 \zeta \cos(\omega \sqrt{1-\zeta^2} t) + \frac{1-2\zeta^2}{\sqrt{1-\zeta^2}} \sin(\omega \sqrt{1-\zeta^2} t) \right]$$  \hspace{1cm} (2)

where

$$\omega = 2\pi f$$

$\omega$ resonance frequency [Hz]

$$f$$

$\zeta$ relative damping factor
Then find the peak values of these acceleration responses and plot them on a logarithmic acceleration-vs.-frequency plane to obtain the acceleration response spectrum or as it might be called here, the test response spectrum, TRS.

At last compare the TRS with the RRS and check that the TRS is higher than the RRS in all analyze points.

The SDOF systems of interest normally have resonance frequencies in the range 1-45 Hz spaced 1/6 octave apart and a relative damping factor of 5%. This gives a total of 34 SDOF systems for which the acceleration response must be calculated. This can be done in at least three fundamentally different ways: (1) convolution in the time domain between the acceleration data obtained from the measurements and the impulse response of the SDOF systems; (2) multiplication in the frequency domain of the Fourier transforms of the above and inverse transformation of the results; (3) sequential calculation of the responses under the assumption of an excitation consisting of frequencies below the Nyqvist frequency. The last method has been chosen here. To get a high resolution in the determination the signals are interpolated to get a at least 20 sampling points at each fundamental period of the SDOF system responses. It can be shown that this method will give an error less than 2 percent. The calculated values will always be too small.

5.2 Transfer function estimation

The vibrator table excitation and the responses at the test object are sampled by the data acquisition system. The transfer functions are obtained by Fast Fourier Transform, FFT, technique. The signals are then broken into overlapping sections and estimates of the transfer functions are obtained as averages of periodograms of these sections modified by a Hanning window. This is a good overall purpose weighting function for continuous signals. The section length is typically 1024 points and the overlap 2/3. With this overlap an effective flat time weighting is achieved.

When the transfer function has been calculated, the modal parameters can be determined by a curve fitting procedure. The transfer function as a sum of contributions from Single Degree of Freedom Systems is given by

$$H(f) = \sum_{i=1}^{k} C_i \cdot \left( \frac{f_i^2 + i \cdot 2 \zeta_i f_i}{f_i^2 - f^2 + i \cdot 2 \zeta_i f_i f} \right),$$

(3)

where

- $H$: Transfer function
- $f$: Frequency [Hz]
- $C_i$: Modal constant
- $f_i$: Resonance frequency [Hz]
- $\zeta_i$: Relative damping

The values of the modal constant, the resonance frequency and the relative damping giving the best curve fits to the magnitude of the transfer function are obtained by the
least square method. This adaptation is done in a frequency range around the resonance frequency. The size of this range has to be chosen manually. Different ranges can give somewhat different values of the modal parameters. However, if the measured transfer function and the theoretic transfer function are plotted in the same graph it is fairly simple to see if the curve fit is good. It is easy to see if the agreement is improved if another frequency range is used. As mentioned in many standards for vibration testing the estimation of damping requires "engineering judgment," the presented damping values should therefore be used with care. Normally $k=1$ is used but if there is a double peak $k=2$ is used

6. Presentation of results

6.1 The qualification test

Upon requests from the customer the results can be presented in different ways. The complete result presentation contains figures of recorded acceleration time histories and plots of the analyzed TRS.

![Example of a Time history plot](image)

*Figure 7:* An example of a graph showing a recorded time history. The plotted file and channel are shown in the upper left corner of the graph.
Example of presentation of a TRS analyze

File with TRS: Test66
Channel: VertAccel

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<th>Freq (Hz)</th>
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</table>

Damping = 5%

Figure 8: An example of a graph showing the results from the Test Response Spectrum analysis. The solid line is the RRS and the stars show the TRS. In the top of the table to the right the analyzed file and channel are specified. At the bottom of the table the damping value used at the analysis is given.

6.2 Transfer function estimation

The determined amplitude transfer function is plotted. An example of such a plot is given in Figure 9. Results from the estimation of the modal parameters are given in plots like that in Figure 10.

Example of presentation of a measured transfer function

Figure 9: An example of a graph showing an amplitude transfer function. The analyzed file and channel are shown in the upper left corner of the graph.
Estimated modal parameters

Frequency [Hz]  Amplification [dB]

Estimated modal parameters

Curve fitting between 4.0 Hz and 8.0 Hz gives:
- Modal constant 1.3
- Resonance frequency 5.8 Hz
- Relative damping 6.4%

*Figure 10: An example of a graph showing the modal parameters estimated by curve fitting. The analyzed file and channel are shown in the upper left corner of the graph.*